Inequalities

Rules for solving inequalities

If $A<B$ then $A+C< B+C$
If $A \le B$ then $A+C \le B+C$
If $A<B$ and $C>0$ then $AC < BC$
If $A \le B$ and $C >0$ then $AC \le BC$
If $A<B$ and $C<0$ then $AC > BC$
If $A \le B$ and $C <0$ then $AC \ge BC$

Note when $C<0$ the inequality switches direction.

Given $A>0$ and $B>0$
If $A<B$ then $\frac{1}{A} > \frac{1}{B}$
If $A \le B$ then $\frac{1}{A} \ge \frac{1}{B}$
So for positive numbers, the reciprocals have the inequality direction switched.

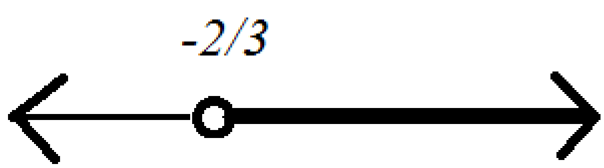
If $A \le B$ and $C \le D$ then $A+C \le B+D$

If $A \le B$ and $B \le C$ then $A \le C$ (transitivity property)

Example: (Solve and Sketch)

$$3x < 9x + 4$$
$$-6x < 4$$
$$6x > -4$$
$$x > -\frac{2}{3}$$

$(-2/3, \infty)$ or $\{x : x > -2/3\}$



Example:

$$4 \le 3x - 2 < 13$$
$$6 \le 3x < 15$$
$$2 \le x < 5$$

$[2,5)$ or $\{x : 2 \le x < 5\}$

Non-Linear Inequalities

Example:

$$x^2 - 5x \le -6$$
$$x^2 - 5x + 6 \le 0$$

Solve the equation $x^2 - 5x + 6 = 0$ and find the **critical points**

$$x^2 - 5x + 6 = 0$$
$$(x-2)(x-3) = 0$$

The points are 2 and 3.

Check the points and the regions they divide the real number line into.

$$x^2 - 5x \le -6$$
$$x^2 - 5x + 6 \le 0$$

$x<2$, try $x=0$	$0 \le -6$	False
$x=2$	$-6 \le -6$	True
$2<x<3$, try $x=5/2$	$-6\frac{1}{4} \le -6$	True
$x=3$	$-6 \le -6$	True
x>3, try $x=4$	$-4 \le -6$	False

So the solution is $[2,3]$ or $\{x : 2 \le x \le 3\}$

Example:

$x(x-1)^2(x-3)<0$

The critical points are 0, 1,3

-1	0<8	False
0	0<0	False
1/2	-5/16<0	True
1	0<0	False
2	-2<0	True
3	0<0	False
4	36<0	False

$(0,1)$ *and* $(1,3)$ or $\{x: 0<x<1 \text{ } or \text{ } 1<x<3\}$

Inequality with a Rational Expression

$$\frac{1+x}{1-x} \geq 1$$

Note: $x \neq 1$

$$\frac{1+x}{1-x} \geq 1$$

$$\frac{1+x}{1-x} - 1 \geq 0$$

$$\frac{1+x-(1-x)}{1-x} \geq 0$$

$$\frac{2x}{1-x} \geq 0$$

Critical points are at 0 and 1

-1	$-1 \geq 0$	False
0	$0 \geq 0$	True
1/2	$2 \geq 0$	True
1	Undefined	False
2	$-4 \geq 0$	False

$[0,1)$ or $\{x : 0 \leq x < 1\}$