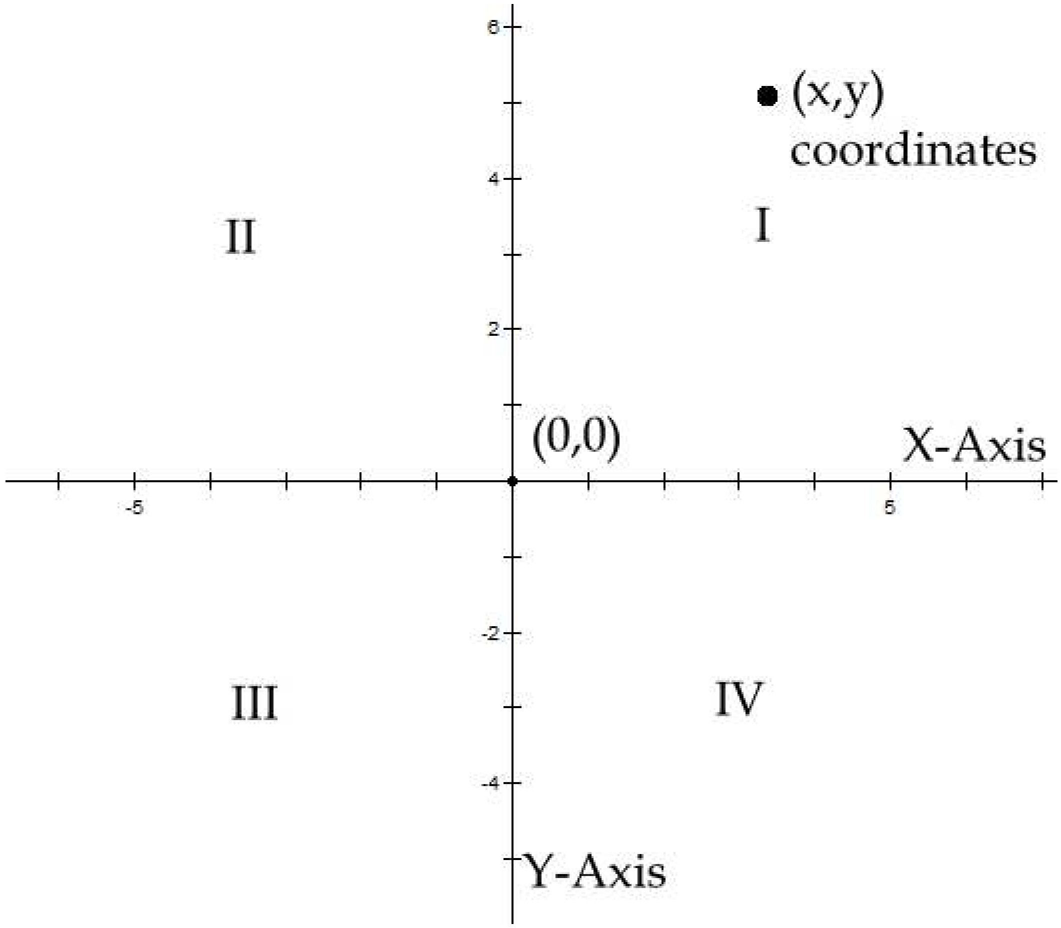
Section 1-9, Mathematics 108

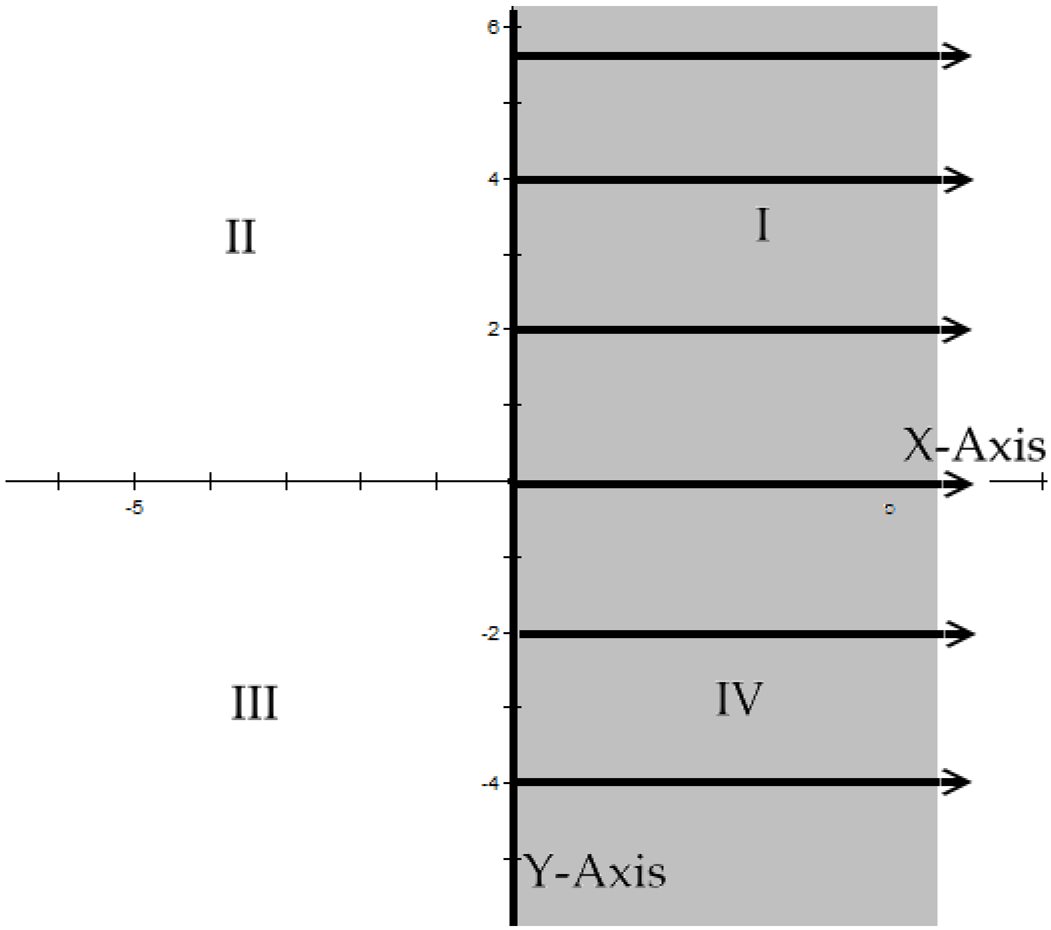
The Coordinate Plane



Note I, II, III and IV are called **quadrants**.

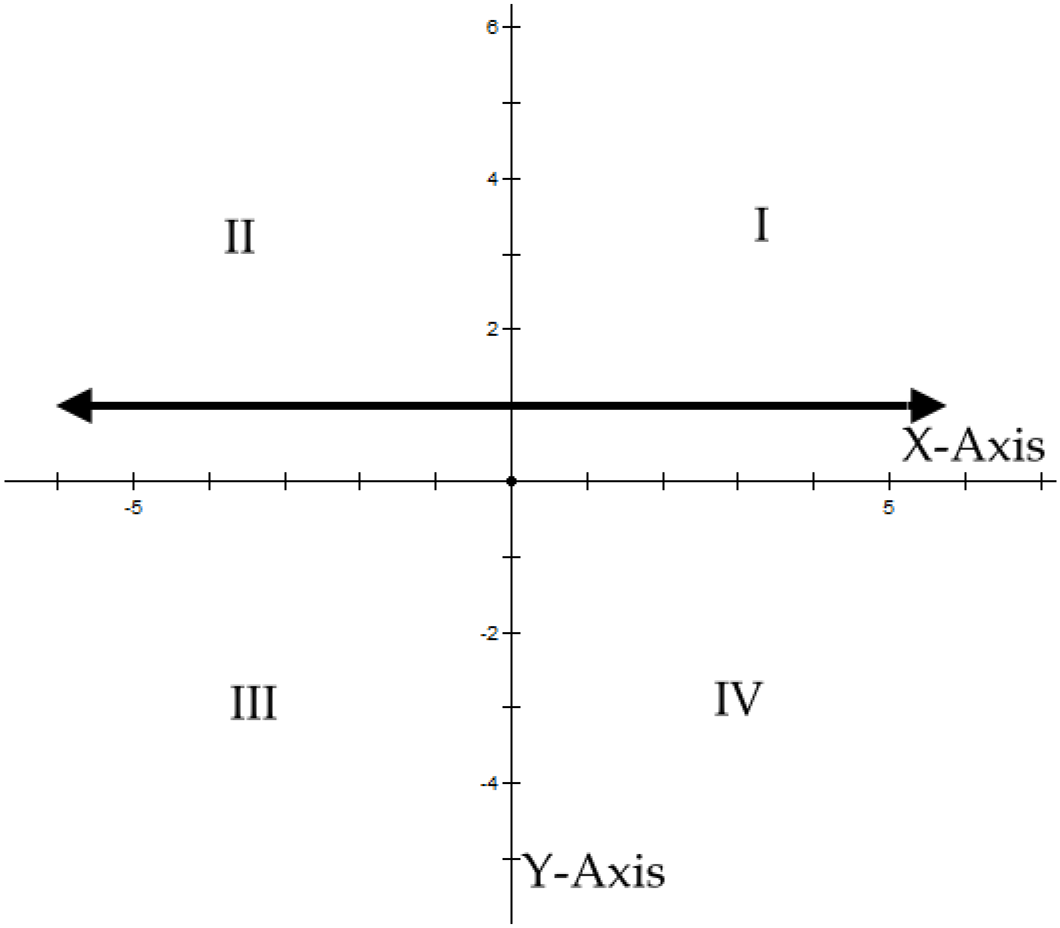
To graph a region

$\{(x, y) : x \geq 0\}$



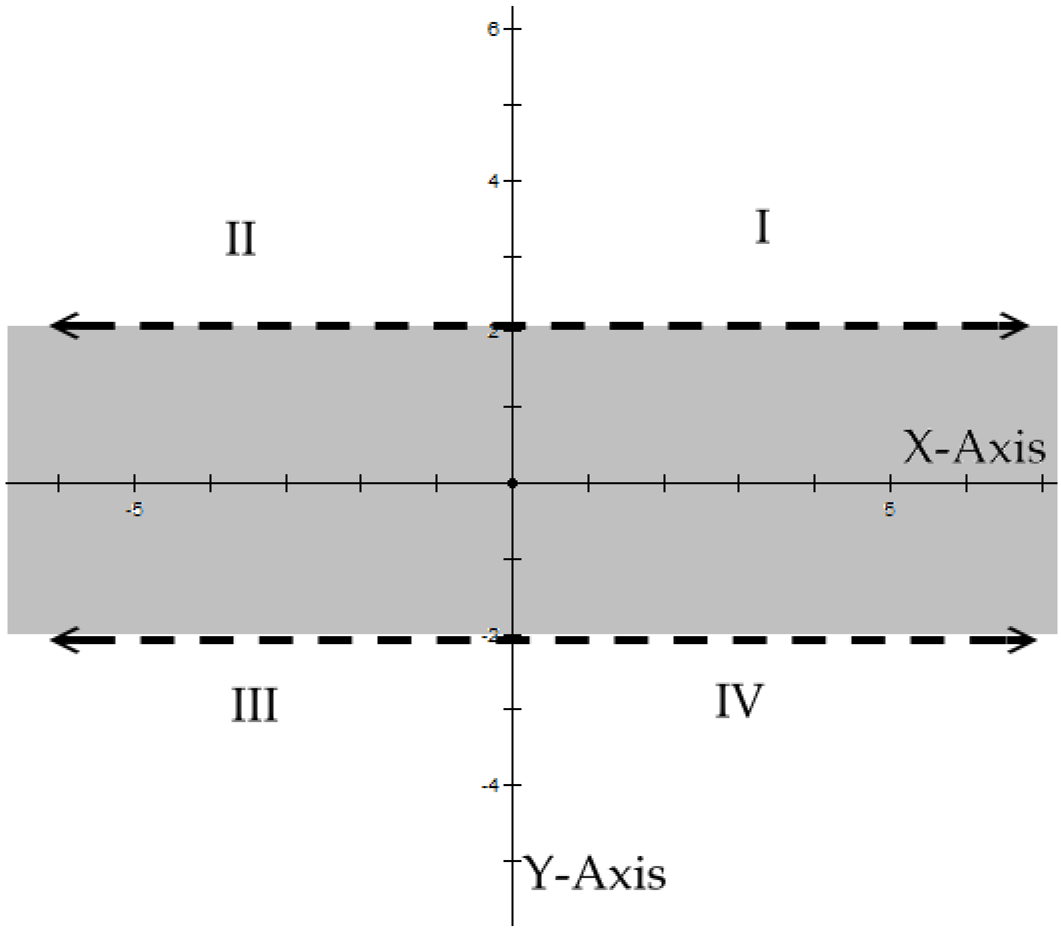
To Graph a horizontal line

$\{(x, y) \mid y = 1\}$



Another Region

$\{(x,y) \mid |y| < 2\}$



Distance Between Two Points

Label two points $A(x_1, y_1)$ *and* $B(x_2, y_2)$

Using the lengths of the legs of the right triangle,

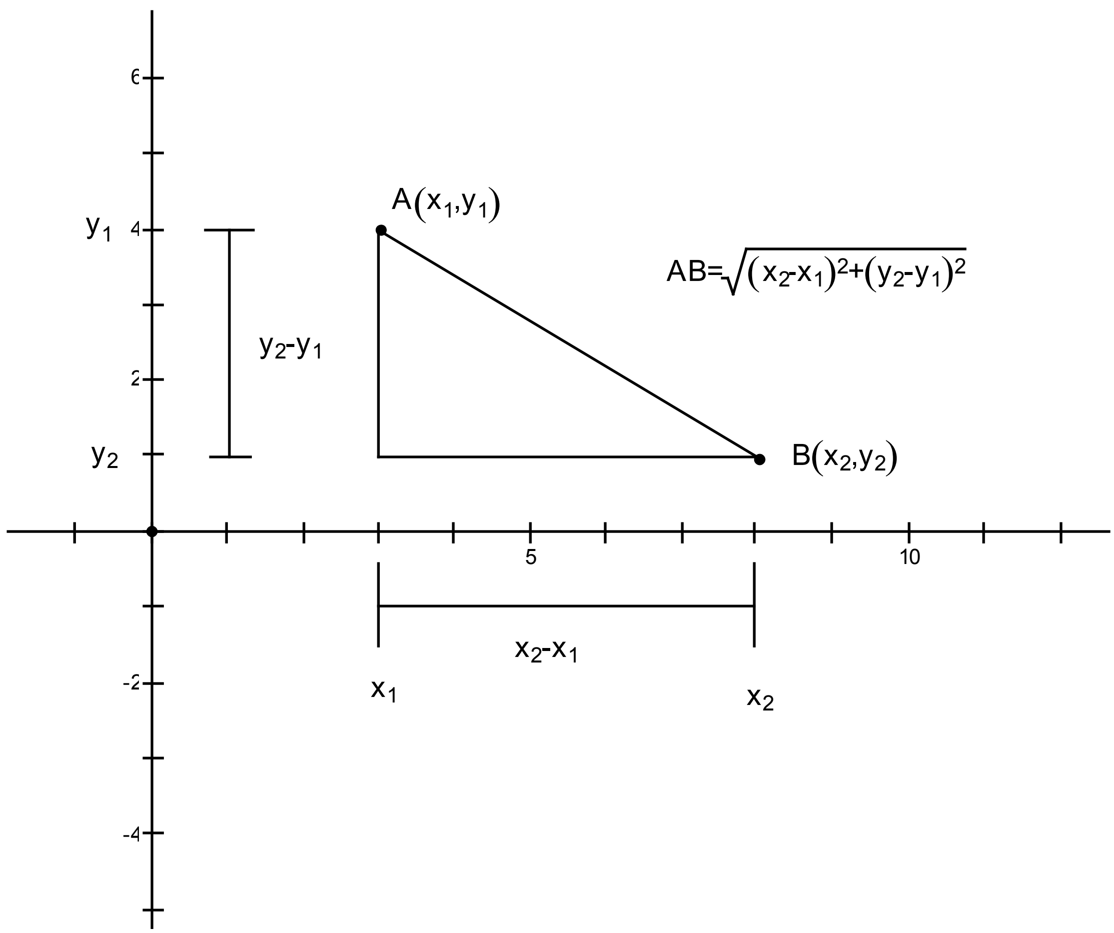
$|x_2 - x_1|$ *and* $|y_2 - y_1|$

And the Pythagorean Theorem

we see that
$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

or that the length of AB *is*

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example:

Find the distance between $(5,7)$ *and* $(2,3)$

$$D=\sqrt{(5-2)^2+(7-3)^2}=\sqrt{9+16}=\sqrt{25}=5$$

Example:

Which point $A(1,-2)$ *or* $B(8,9)$ is closer to $C(5,3)$

$$AC=\sqrt{(5-1)^2+(3-{}^-2)^2}=\sqrt{16+25}=\sqrt{41}$$
$$BC=\sqrt{(5-8)^2+(3-9)^2}=\sqrt{9+36}=\sqrt{45}$$

$\sqrt{41}<\sqrt{45}$ so AC is closer.

The Midpoint Formula

For points $A\left(x_1, y_1\right)$ *and* $B\left(x_2, y_2\right)$ the mid point is

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

To prove this use the distance formula to show that *AM=BM=AB/2*.

$$AM = \sqrt{\left(x_1 - \frac{x_1+x_2}{2}\right)^2 + \left(x_1 - \frac{x_1+x_2}{2}\right)^2} =$$

$$\sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2} = \frac{\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}}{2} = \frac{AB}{2}$$

Similarly

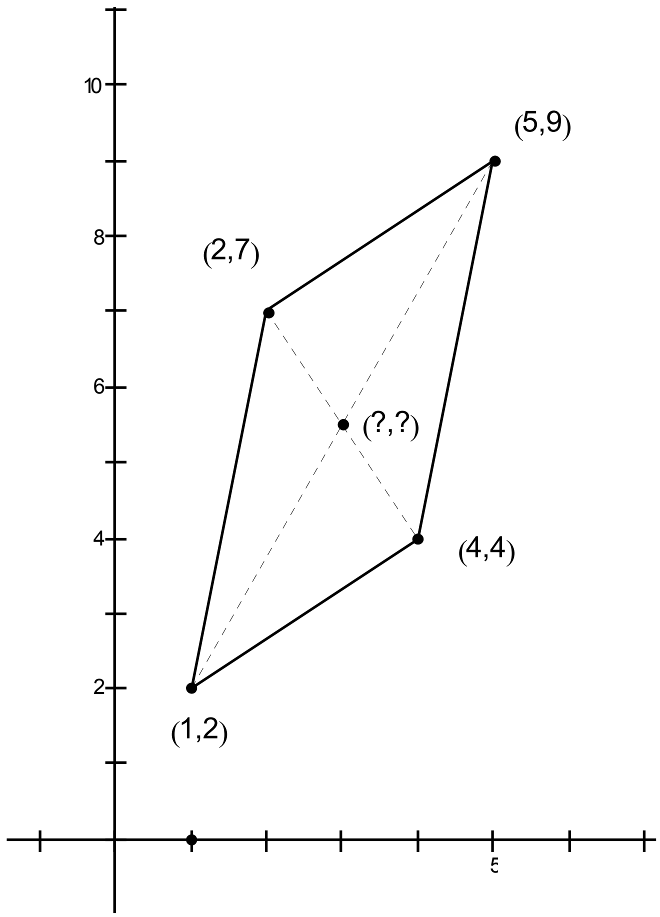
$$BM = \frac{AM}{2}$$

Example:

What is the midpoint of $(9,3)$ *and* $(-3,5)$

$$M = \left(\frac{9+{}^{-}3}{2}, \frac{3+5}{2}\right) = (3,4)$$

Example:



Given the four points $(1,2),(2,7),(5,9)$ *and* $(4,4)$
prove that they are the vertices of a parallelogram by showing that their diagonals bisect each other.

Do this by showing they have the same midpoint.

$$MP_1 = \left(\frac{2+4}{2},\frac{7+4}{2}\right) = \left(3,\frac{11}{2}\right)$$

$$MP_2 = \left(\frac{1+5}{2},\frac{2+9}{2}\right) = \left(3,\frac{11}{2}\right)$$

So Yes! a Parallelogram.

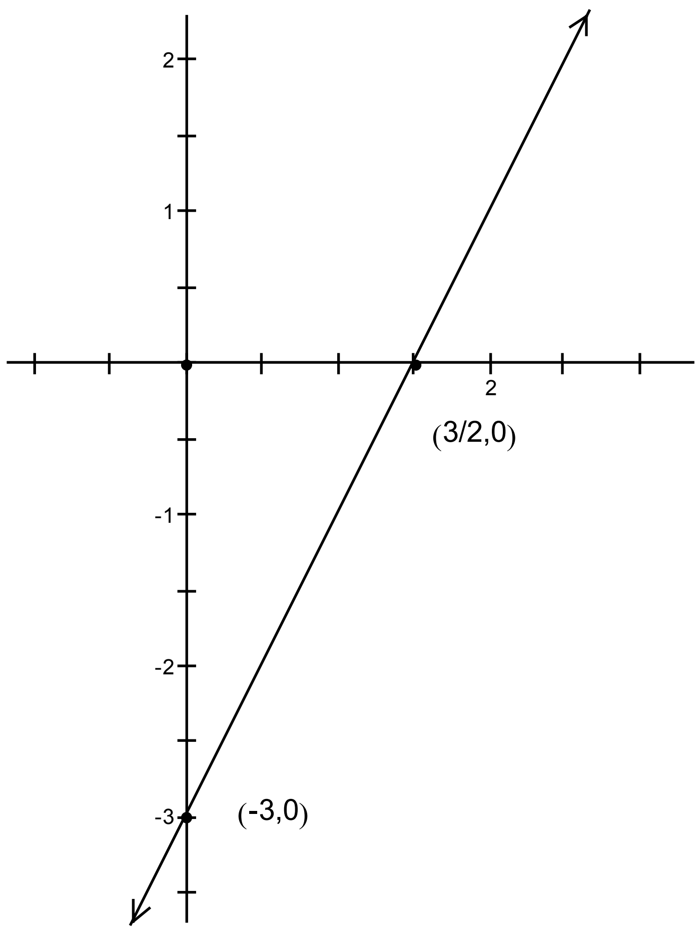
Graphing a line

Recall from geometry that 2 points determine a line so

if $2x - y = 3$

We can pick x=0 and get y=-3

and we can pick y=0 and get x=3/2



These points are called the X and Y intercepts because they cross the X and Y axes.

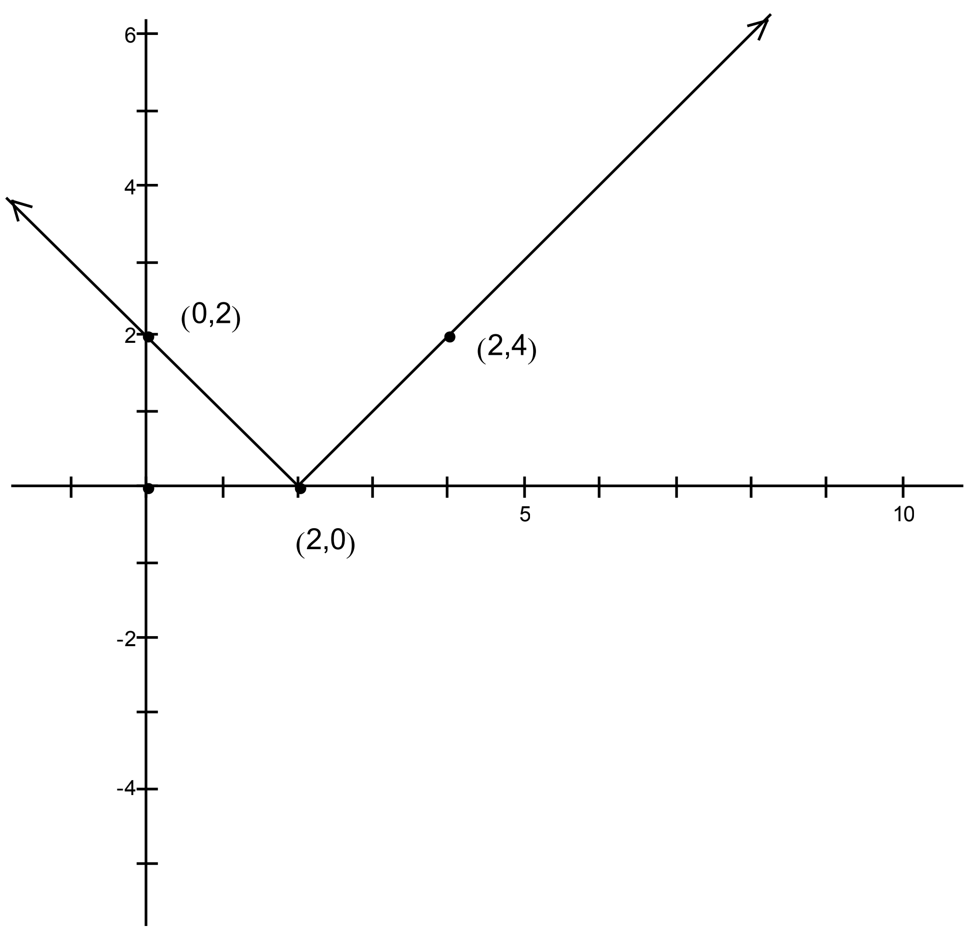
Graphing an absolute value equation

Let $y = |x-2|$

Note that at *x*=2 there is a **critical point** where the graph will change direction.

Let's pick 3 points including the critical point, one before it and one after:

$x = 2 \quad (2,0)$
$x = 0 \quad (0,2)$
$x = 4 \quad (4,2)$

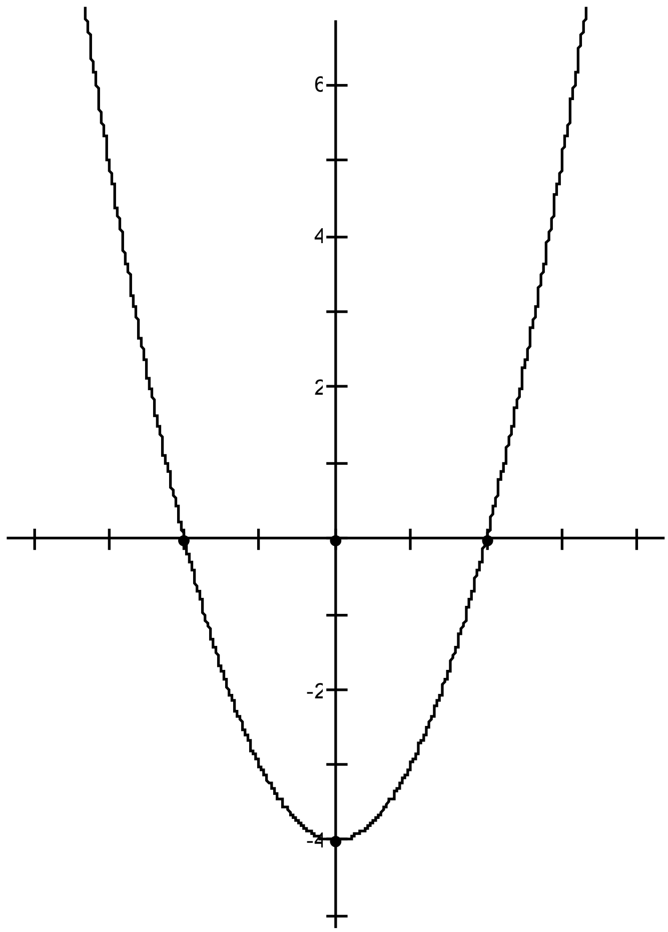


Graphing a parabola using intercepts

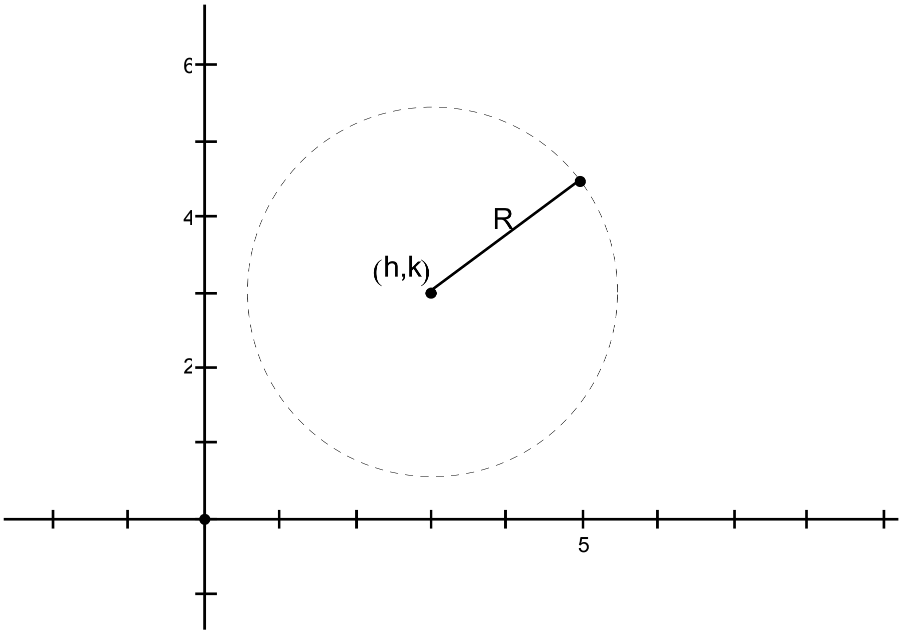
Let $y = x^2 - 4 = (x+2)(x-2)$

The two x intercepts are at x={1,-2}

The y intercept is at -4



Graphing a Circle



Let a circle be centered at (*h,k*) with radius *R*.

Then if (*x,y*) is a point on the circle, we can use the distance formula:

$$D\left[(h,k),(x,y)\right] = R$$
$$\sqrt{(x-h)^2 + (y-k)^2} = R$$

Squaring both sides we get the equation of a circle:

$$(x-h)^2 + (y-k)^2 = R^2$$

Example:

Find the equation of a circle centered at (2,-5) with radius 3

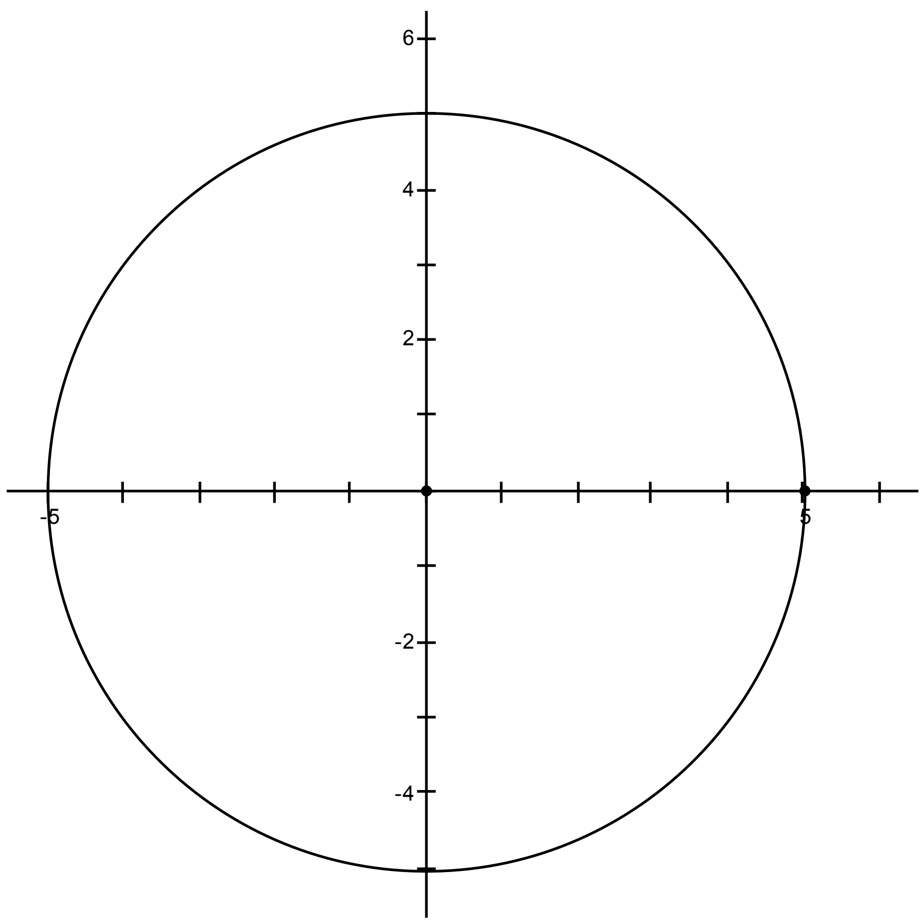
$$(x-2)^2+(y-{}^{-}5)^2=3^2$$
$$(x-2)^2+(y+5)^2=9$$

Example:

Graph the equation

$$x^2+y^2=25$$
$$(x-0)^2+(y-0)^2=5^2$$

So this is a circle centered at (0,0) with radius 5



Example:

(1,8) and (5,-6) are points on the diameter of a circle.
What is the equation of the circle?

Half the distance between the points will be the radius:

$$R = \frac{\sqrt{(1-5)^2 + (8-{}^{-}6)^2}}{2} = \frac{\sqrt{16+196}}{2} = \frac{\sqrt{212}}{2} = \sqrt{53}$$

The center will be at the midpoint of the two points:

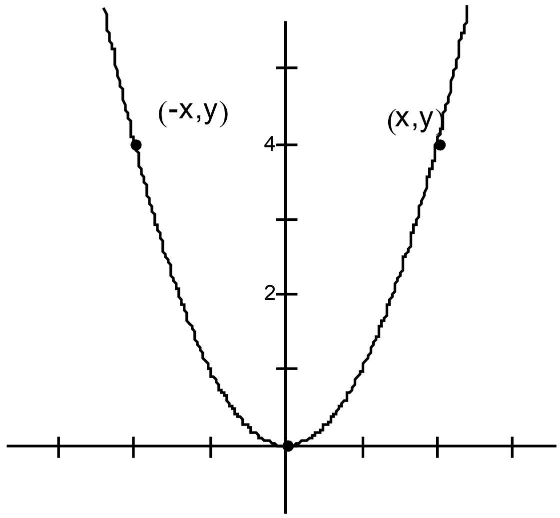
$$(h,k) = \left(\frac{1+5}{2}, \frac{8+{}^{-}6}{2}\right) = (3,1)$$

So the equation will be

$$(h-3)^2 + (k-1)^2 = \left(\sqrt{53}\right)^2 = 53$$

Symmetry in Graphs

Note that the parabola $y = x^2$ is symmetric with respect to the Y-axis.

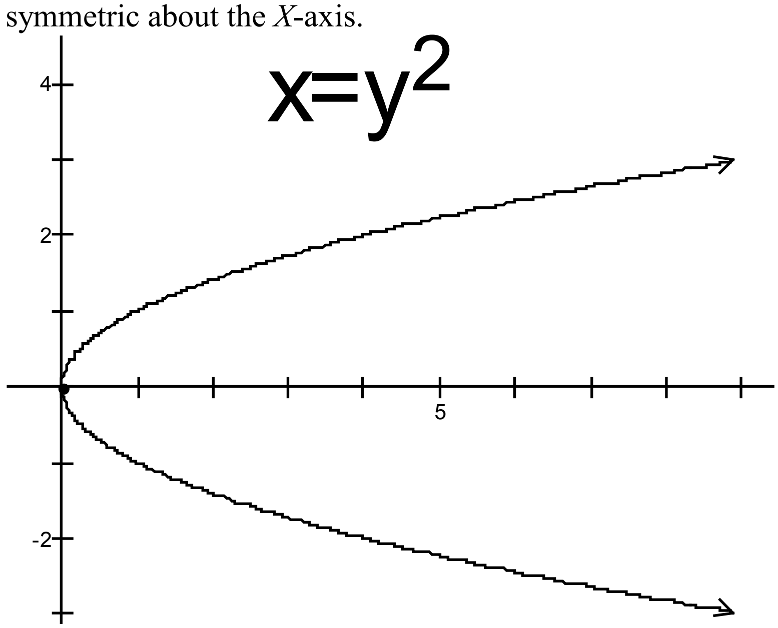


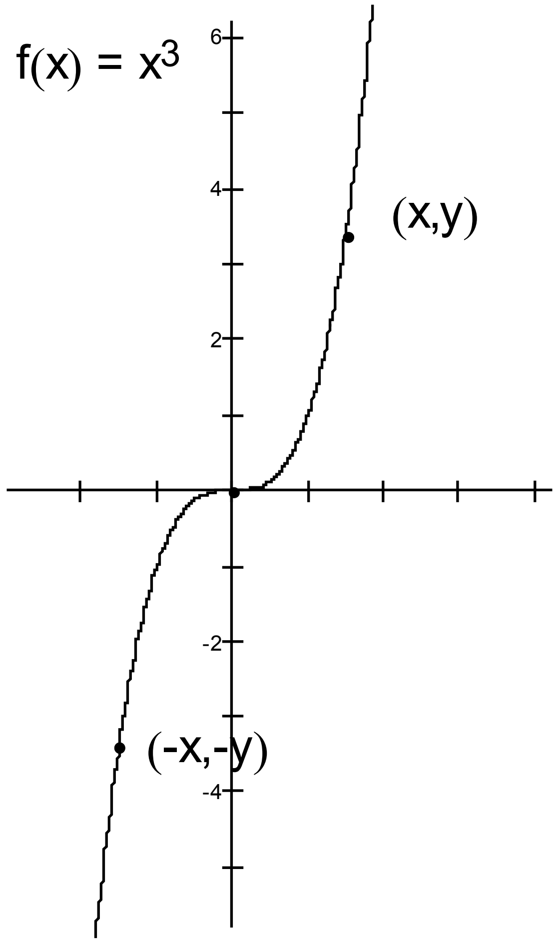
Note that for every point (*x,y*) on the graph, the point (-*x,y*) will also be on the graph.

So if we substitute -*x* for *x* in the equation, we get the same equation:

$$y = (-x)^2 = x^2$$

If you can substitute -*y* for *y* without changing the equation, then the graph will be symmetric about the *X*-axis.





If (*x,y*) being on the graph means that (-*x,-y*) then substituting -*x* for *x* and -*y* for *y* will not change the equation, and the graph is symmetric about the origin.

Example:

Check this equation for symmetry.

$y = x^3 - 9x$

Substituting -*x* for -*x* gives:

$y = (-x)^3 - 9(-x) = -x^3 + 9x$

So the graph is not symmetric around the *Y*-axis.

Substituting -*y* for -*y* gives:

$-y = x^3 - 9x$

$y = -x^3 + 9x$

So the graph is not symmetric around the *Y*-axis.

Substituting -*x* for -*x* and -*y* for -*y* gives:

$-y = (-x)^3 - 9(-x) = -x^3 + 9x$

$y = x^3 - 9x$

So the graph will be symmetric about the origin: