

## Periodic Functions

### Formal Definition

A function  $f$  is periodic if  $f(x + p) = f(x)$  for all  $x$  in the domain of  $f$  where  $p$  is a positive constant called the **period** of the function.

Note that if a function has period  $p$  it also will have period  $2p$ ,  $3p$ , etc.  
In general we will be interested in the smallest or shortest period.

One way to think about a periodic function, is that it is a function that you can do a transformation on the function along the  $x$  axis and end up with the same function.

### Examples of periodic functions:

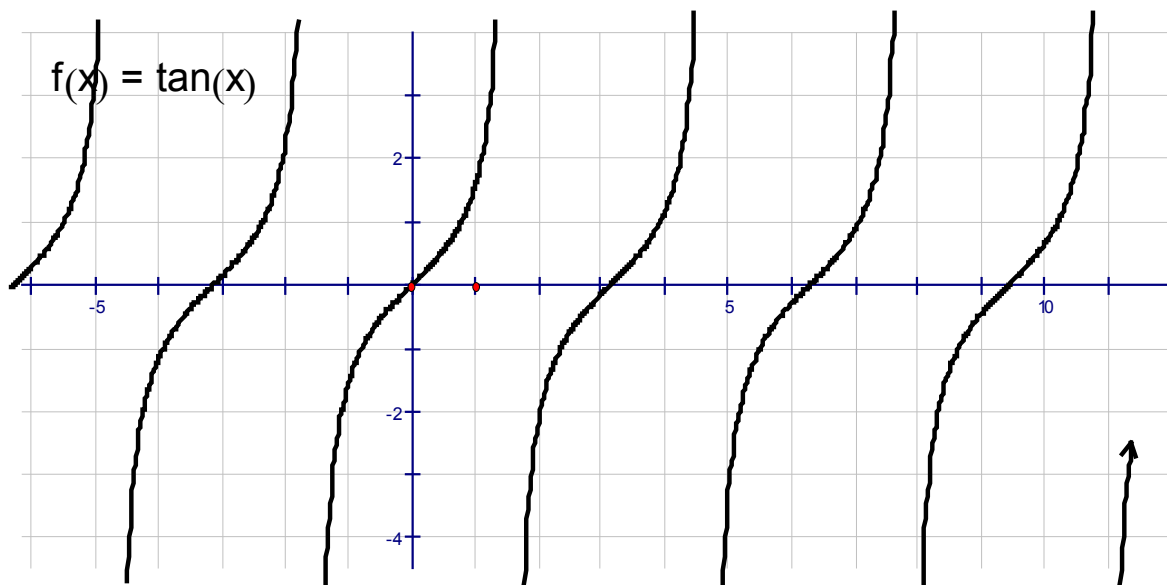
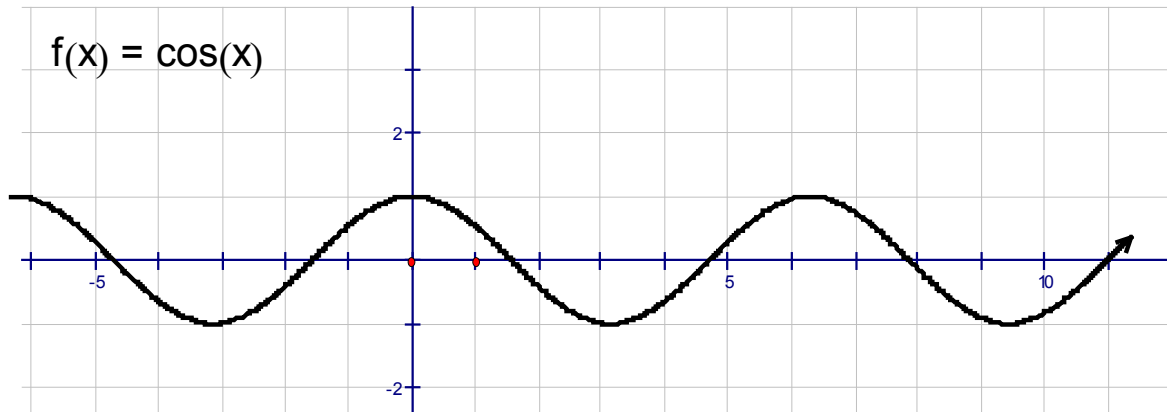
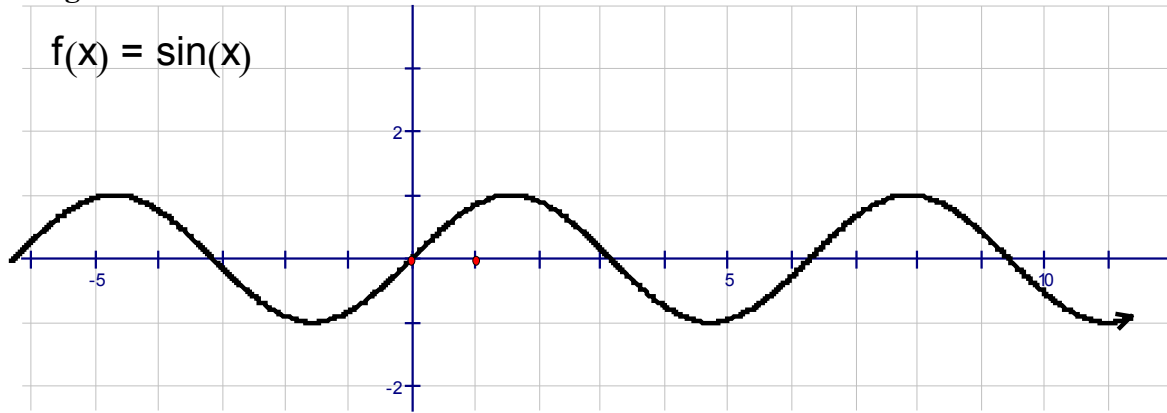
The time on a clock.

The ocean tides.

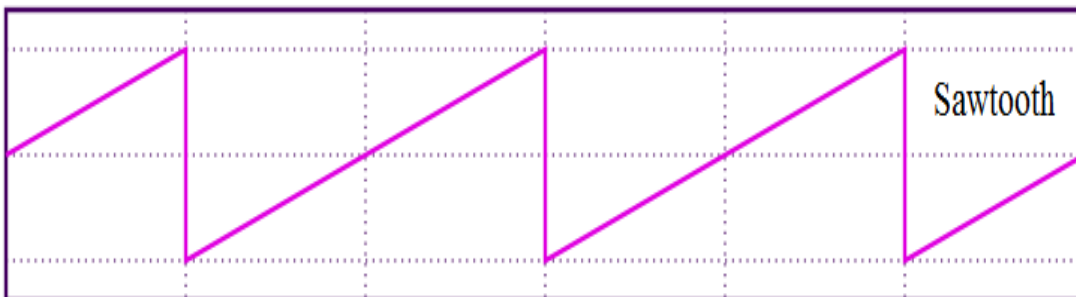
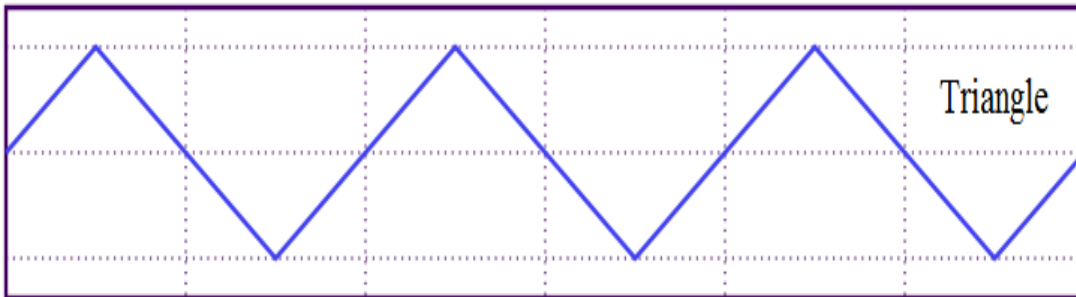
The position of the earth around the sun.

Note that many periodic functions are functions of time.

## Trig Functions



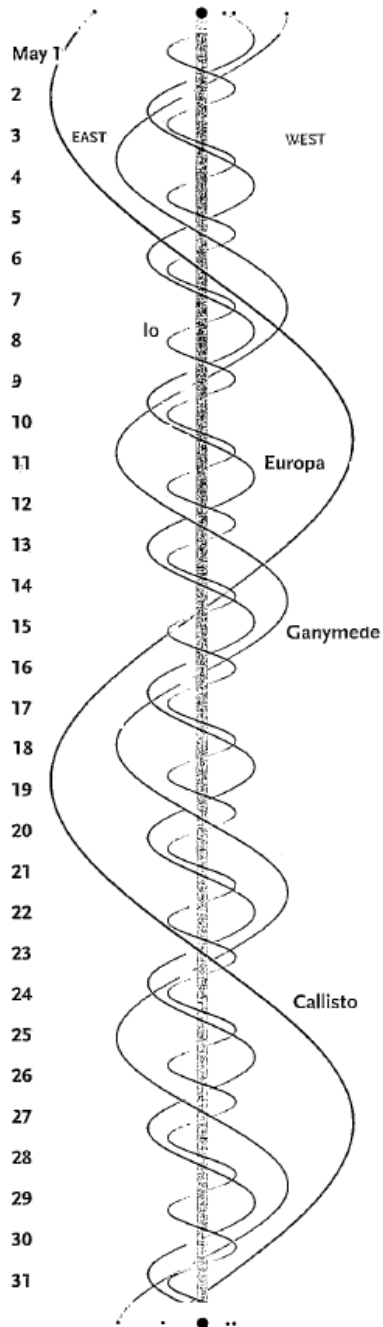
**Square, Triangle and Sawtooth waves**



# Examples of phenomenon from the real world that are modeled by periodic functions

Orbits of planets and moons:

## Jupiter's Moons



The wavy lines represent Jupiter's four big satellites. The central vertical band is Jupiter itself. Each gray or black horizontal band is one day, from 0<sup>h</sup> (upper edge of band) to 24<sup>h</sup> UT (GMT). UT dates are at left. Slide a paper's edge down to your date and time, and read across to see the satellites' positions east or west of Jupiter.

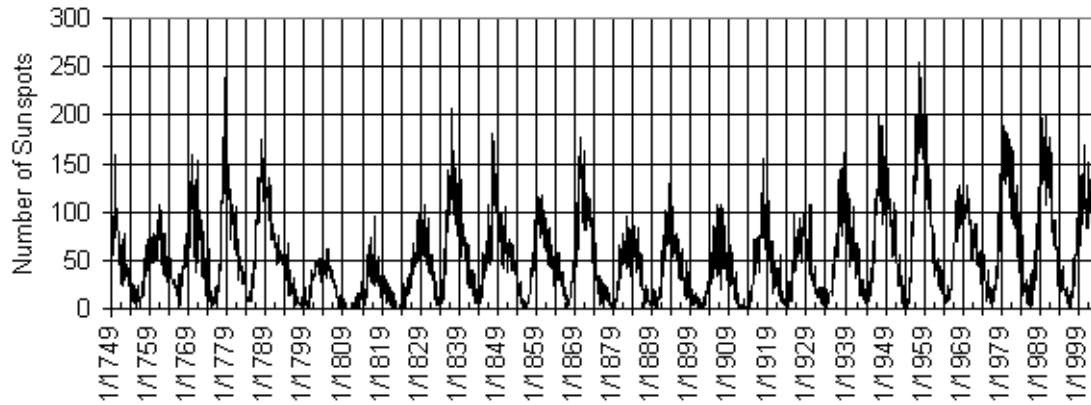
## Sun Spots

### Sunspot Cycles 1749 - July 2003

Average Monthly Count

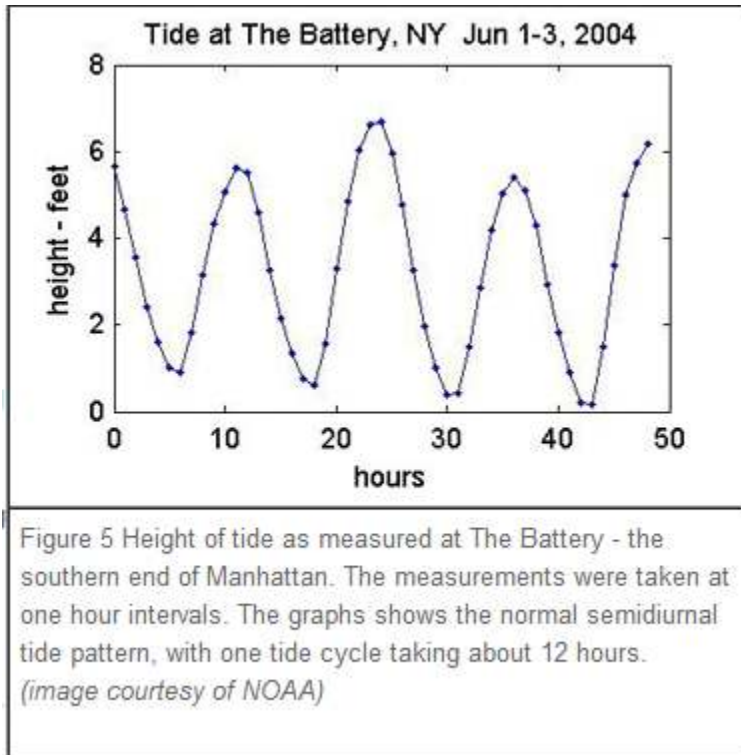
source: International Sunspot Count (Brussels SIDC Index)

— Sunspot Monthly Average

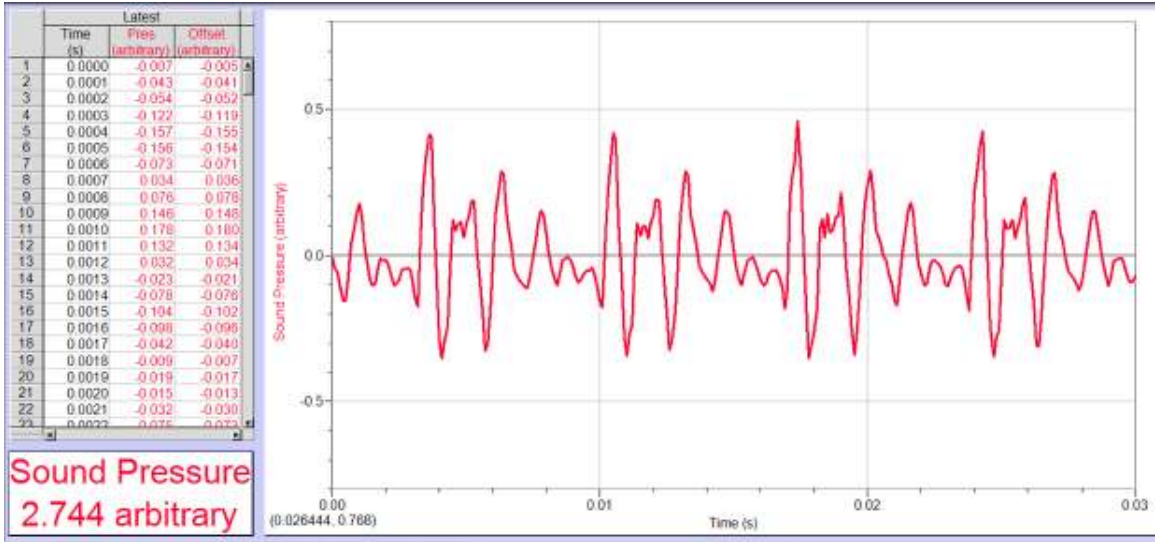


Compiled from the International Sunspot Count by MWM, 2002. Most major human conflicts are easily correlated to the peaks in the count of sunspots and so are major weather disturbances, including severe droughts and floods. [MichaelMandeville.com](http://MichaelMandeville.com)

## Tides



# Sound Waves



<http://www.schoenbrun.com/foothill/math48c-2/mpeg/SoundWaves-2.00.mp4>

## Important features of periodic functions

### The Amplitude

The amplitude of a periodic function is defined as

$$y = \frac{\text{max of } f - \text{min of } f}{2}$$

### Frequency

For a time based function, the period has the units of time. In this case we sometimes call the period a **cycle**.

The period then can be described as cycles/time or for example cycles per second.

The reciprocal of a time based function is called the **frequency** and has the units cycles/time, for example 1 cycle per second.

We will use the term frequency whether for a periodic function whether it is time based or not.

### Range

Just like any other kind of function, the range of a periodic function is the set of all values a function can have.

Question: What is the period, frequency, amplitude and range of this sine function?

