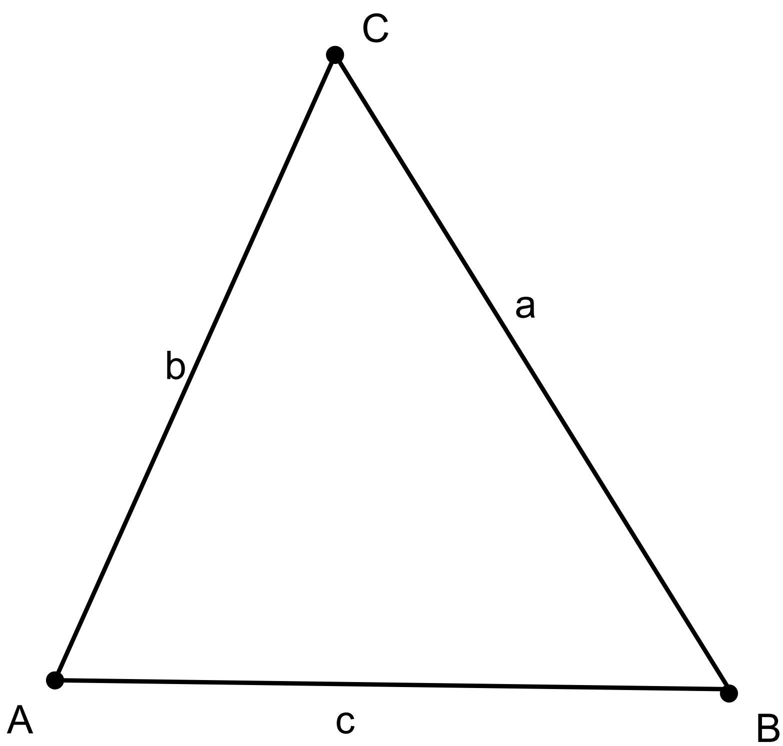
Trigonometry 10 Mathematics 108

Law of Cosines

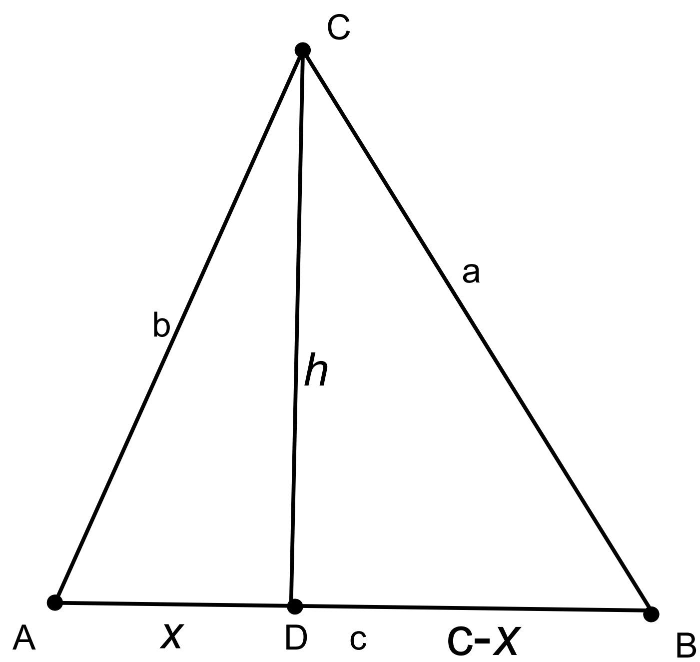


Given that we know the sides a, b and c, how can we find the angles?

Similarly, what if we know side's b and c and angle A?

Show DVD Law O fConsines

Deriving the law of sines:
Drop an altitude length h, dividing c into x and (c-x)



Note the following trigonometric relationship:

$$\cos(a) = \frac{x}{B}$$

Multiplying by B we get

$B\cos(a) = x$ We will use this later

Now use the Pythagorean theorem on the two right triangles getting

$$B^2 = h^2 + x^2$$

and

$$A^2 = h^2 + (C - x)^2$$

solve both equations for h^2

$h^2 = B^2 - x^2$ and $h^2 = A^2 - (C - x)^2$

by transitivity

$$B^2 - x^2 = A^2 - (C - x)^2$$

Multiply $(C-x)^2$ and simplify

$$B^2 - x^2 = A^2 - (C^2 - 2Cx + x^2)$$

$$B^2 - x^2 = A^2 - C^2 + 2Cx - x^2$$

$$B^2 = A^2 - C^2 + 2Cx$$

And now Solve for A^2

$$A^2 = B^2 + C^2 - 2Cx$$

Now we substitute in x from above and get

$$A^2 = B^2 + C^2 - 2BC\cos(a)$$

Note that we could do this derivation using any permutation of the sides so we also have

$$B^2 = A^2 + C^2 - 2AC\cos(b)$$

and

$$C^2 = A^2 + B^2 - 2AB\cos(c)$$

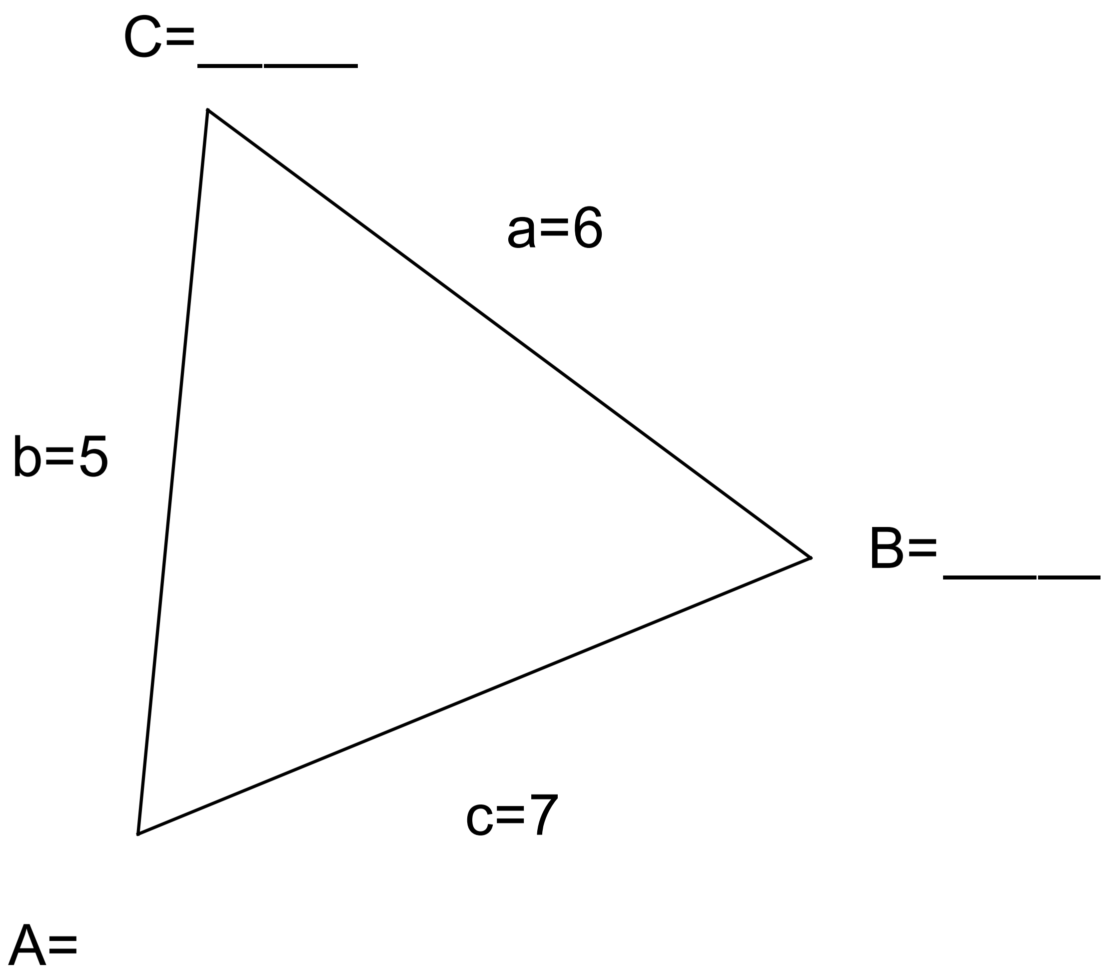
This is the LAW OF COSINES!

$$C^2 = A^2 + B^2 - 2AB\cos(c)$$

Note the choice of A, B and C is arbitrary.

How can we use these?   Let's go back to the previous two unsolved problems

For this problem SSS we use the Law of Cosines



To find the angle across from the 7 side we note that:

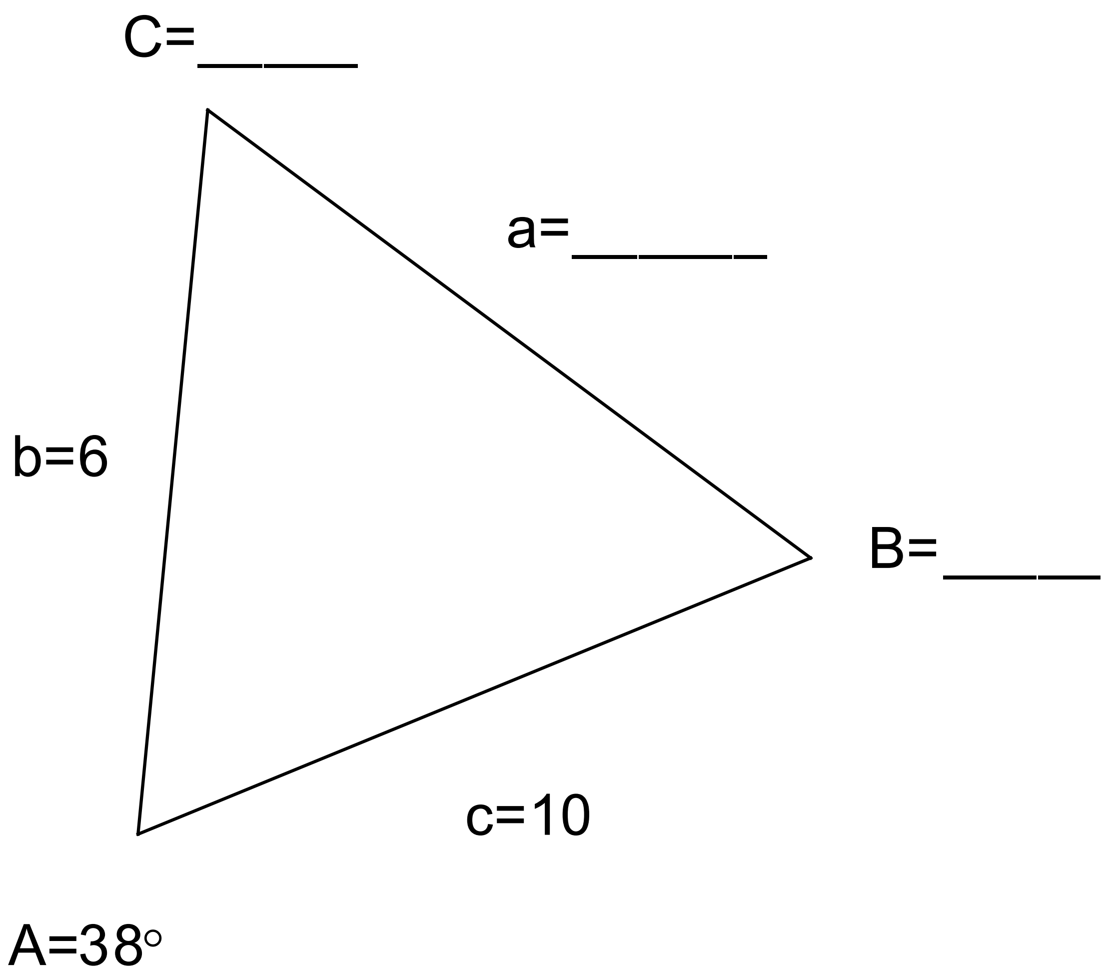
$$7^2 = 5^2 + 6^2 - 2 \cdot 5 \cdot 6 \cos(x)$$

Solving for x we get

$$x = \cos^{-1}\left(\frac{7^2 - 6^2 - 5^2}{-(2 \cdot 5 \cdot 6)}\right) = 78.46^\circ =$$

In the same way we can find the other two angles

For SAS we have the following situation:



Here we use the Law of Cosines to find side a

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$a = \sqrt{6^2 + 10^2 - 120\cos(38^\circ)} \approx \sqrt{135.2} = 11.63$$

Knowing side a we can find the missing sides using the law of sines.

HW: 6.6: 3, 4, 5, 11, 13, 14, 15 25