

Graphs of Sine and Cosine

Take a look at this animation.

<http://schoenbrun.com/foothill/math48c-2/gsp/rCircularMotion2.gsp>

Demonstrate how to graph a function on the calculator

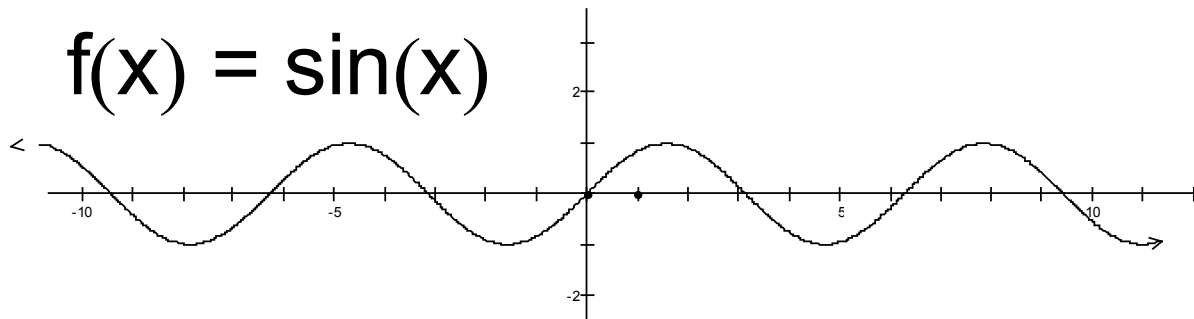
We want to write a very general form of these functions and understand it.

$$f(\theta) = A \sin(B(\theta - C)) + D$$

$$f(\theta) = A \cos(B(\theta - C)) + D$$

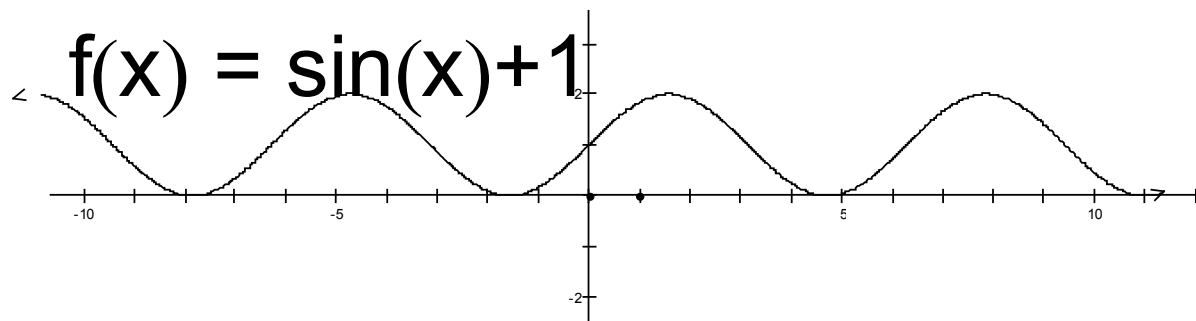
$$f(\theta) = A \sin(B(\theta - C)) + D$$

Start with a simple sine function $A=1, B=1, C=0, D=0!$



$$f(\theta) = \sin(\theta) + D$$

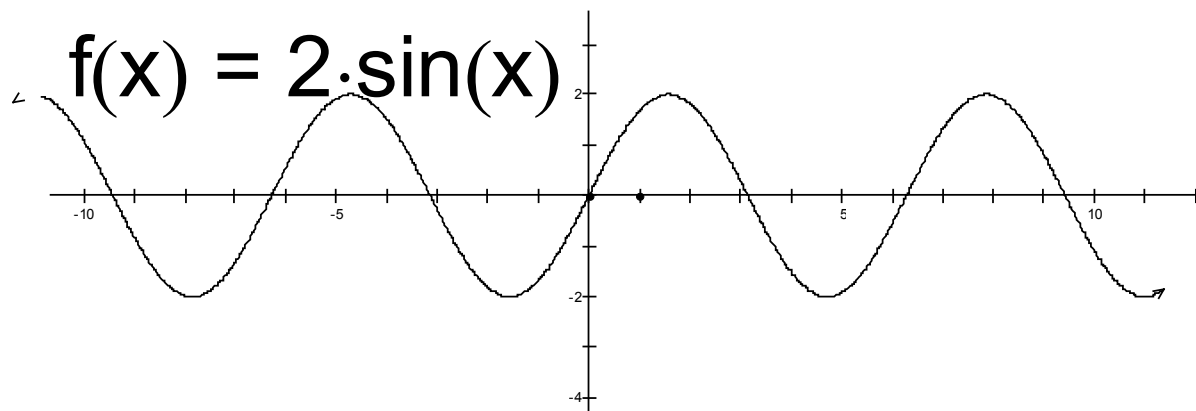
What does having $D \neq 0$ do?



This is of course just a vertical shift transformation you would expect from any function.

$$f(\theta) = A \sin(\theta)$$

What does changing A do?

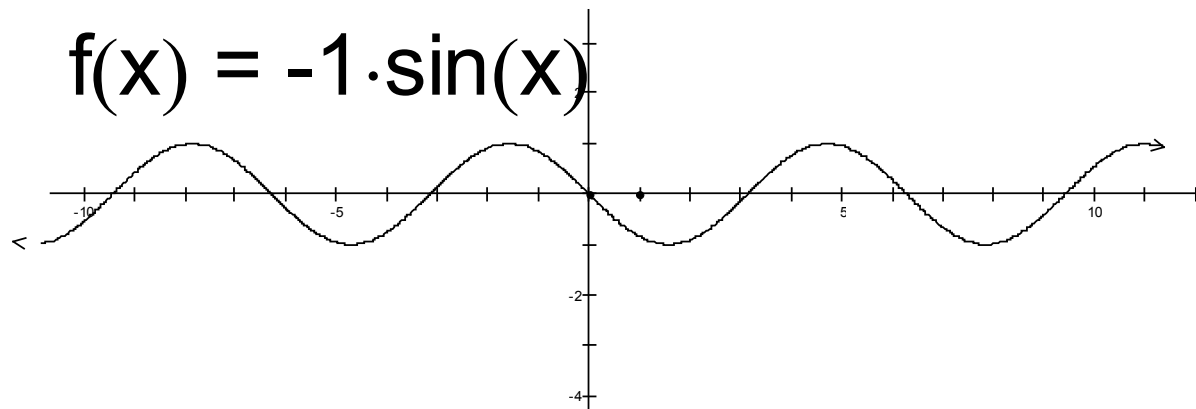


This of course is just a dilation in the Y direction

Notice that $A=2$ is the **Amplitude** of the function.

$$f(\theta) = A \sin(\theta)$$

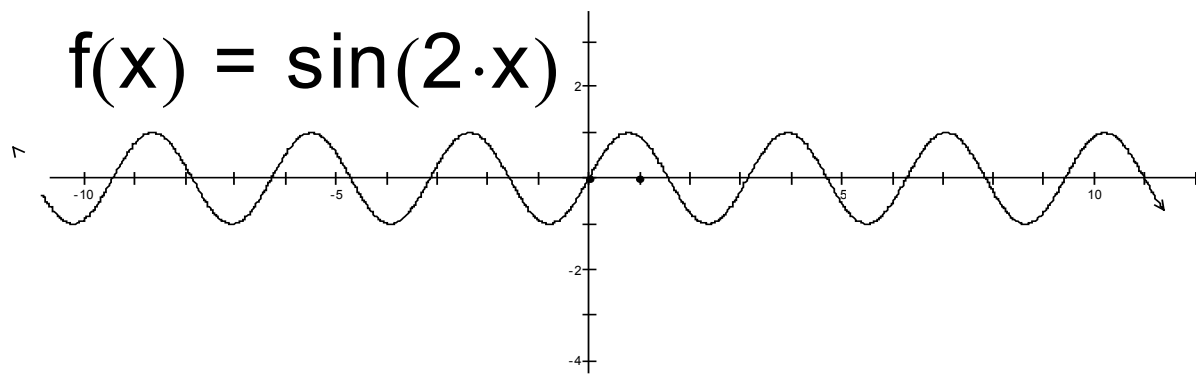
What if A is negative?



This is just a reflection across the x axis.

$$f(\theta) = \sin(B\theta)$$

What does changing B do?

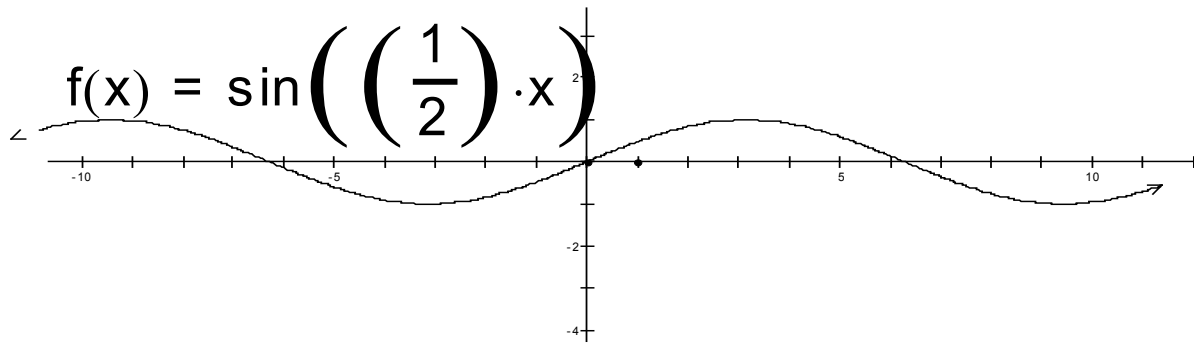


Note that as B gets larger, the period gets smaller.

This is just a dilation in the

$$f(\theta) = \sin(B\theta)$$

What happens when B gets smaller?



When B gets smaller, the period gets larger.

So B changes the PERIOD or the FREQUENCY! Note the inverse relationship to Period.

$$B = 1 \text{ Period} = 2\pi$$

$$B = 2 \text{ Period} = \pi$$

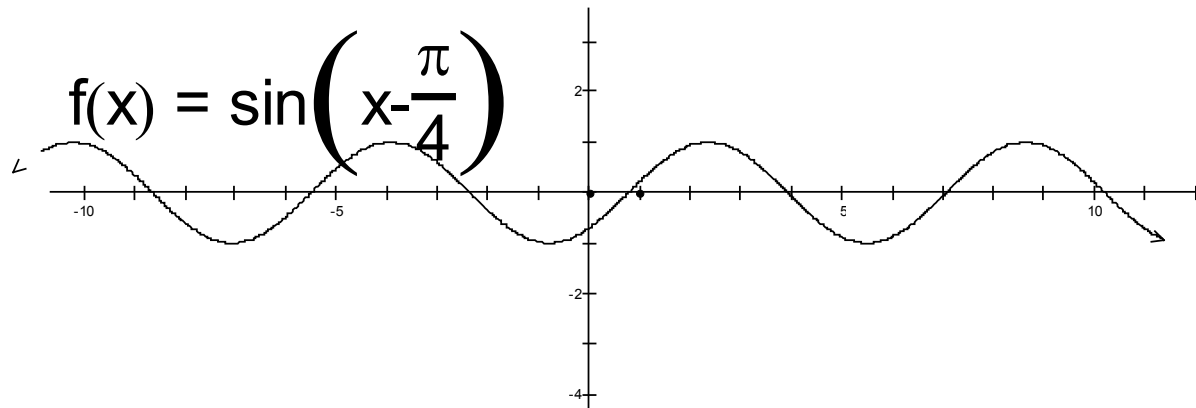
$$B = 1/2 \text{ Period} = 4\pi$$

So the Period of a Sine or Cosine function is $2\pi/B$.

What is the Frequency? Its reciprocal = $B/2\pi$

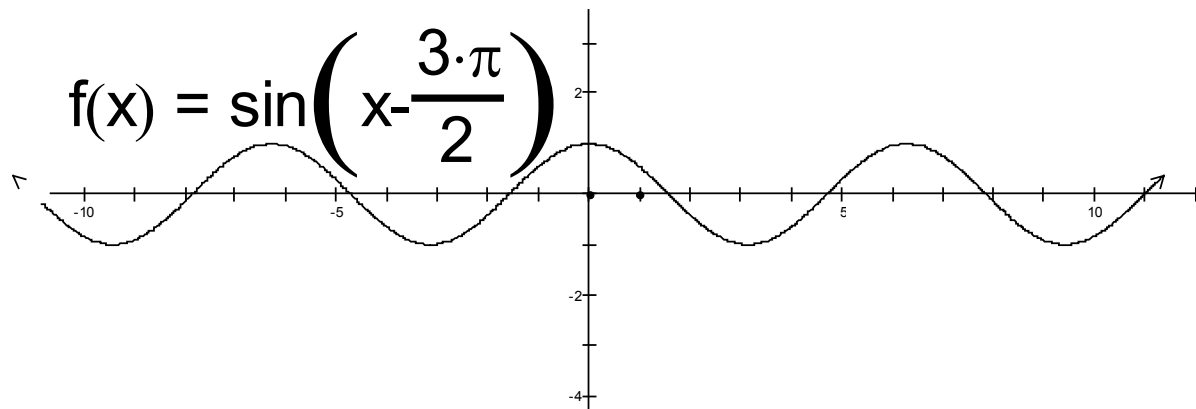
$$f(\theta) = \sin(\theta - C)$$

Finally what does C do?



Notice the starting point $(0,0)$ has now moved to the right $(\pi/4, 0)$. This is a horizontal translation. It is also known as a horizontal shift. For a sine or cosine function we call this a **PHASE SHIFT!**

Definition: A **phase shift** is the portion of one period shifted horizontally. Note that a Phase shift of $\frac{3\pi}{2}$ of a sine function gives you a cosine function



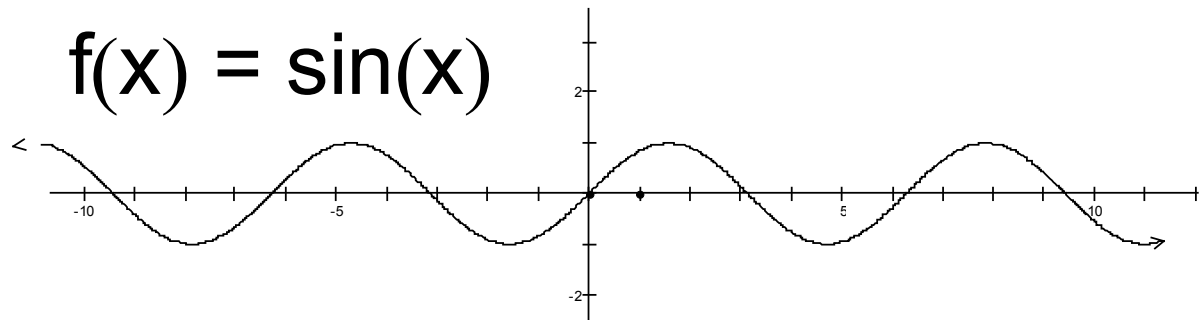
Application: Power in an AC circuit.

US Current is AC 60hz 120 Volts:

Amplitude is 120 Volts, so Voltage varies from +120V to -120V.

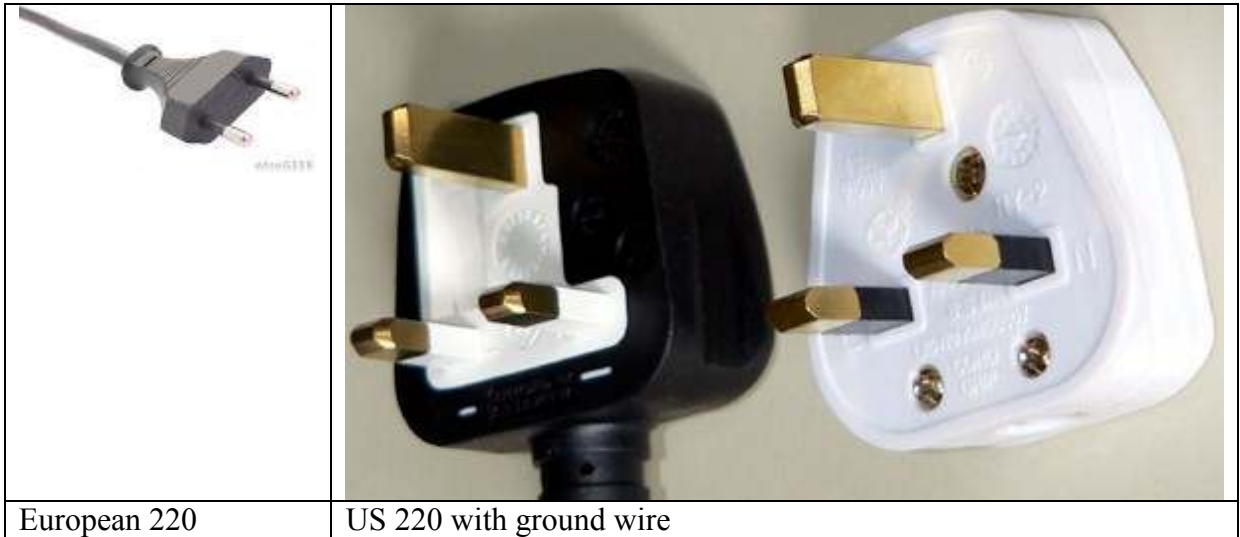
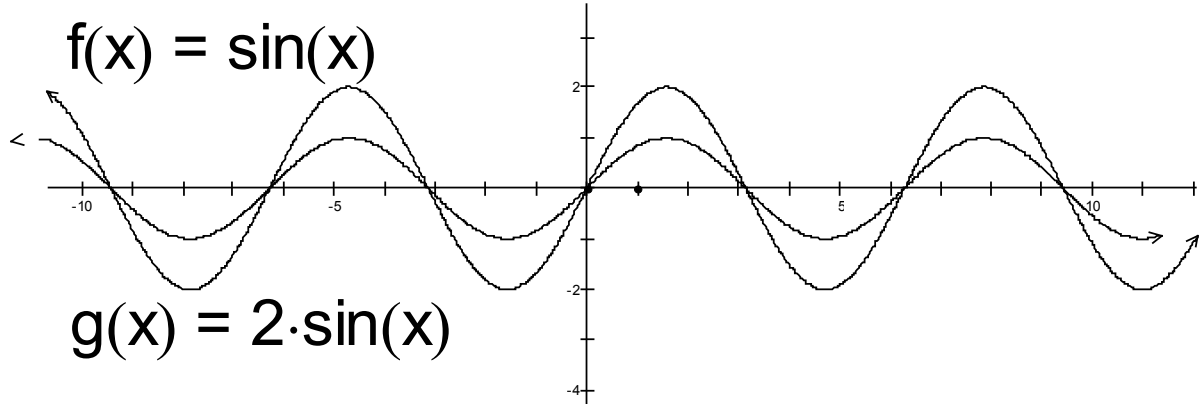
The period is 1/60 second and the frequency is 60hz or 60 times per second

AC uses two wires, with the Voltage between the wires changing over time:



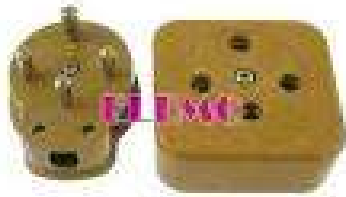
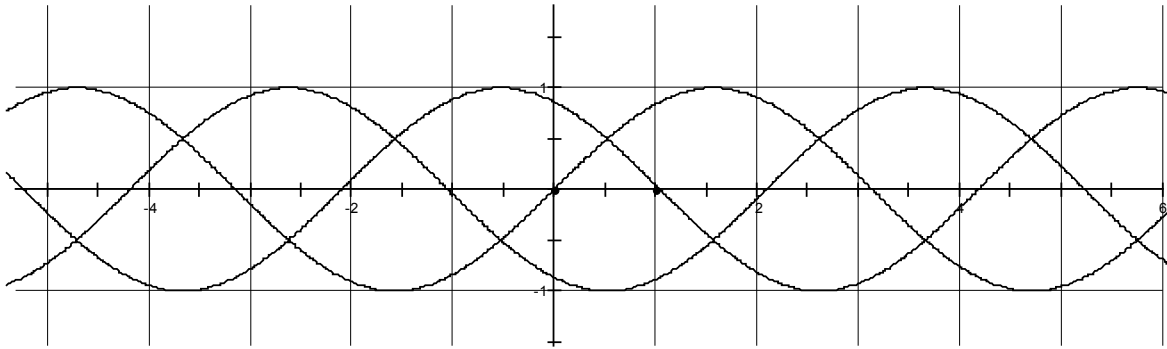
With and without ground wire.

European and some US homes also have 220Volt 60hz AC



Some Canadians have 50hz circuits.

For Industrial Applications, a 4 wire 3 Phase 120 Volt system is used to provide more and smoother power. One ground wire and three hot wires each carry 120V 60hz, but at a phase shift of $1/3$ and $2/3$, or 120° and 240° .



HW: 5.3: 9, 16, 21, 31, 34, 48, 83