Math 109 Calc 1 Lecture 13

Implicit Differentiation

Section 3.5

Until now we have been differentiating only functions, and functions that are described explicitly.

$f(x) = Some\ Algebraic\ Expression\ in\ x$

It is possible that we might have an expression in which the function is only implicitly described, and it may not be a function at all, but a curve which might have a derivative at most points.

An example we will explore is

$x^3 + y^3 = 6xy$

Here it would be difficult if possible, at all to get an expression *y*=*f* (*x*) however it is possible using a technique to find the derivative as a function of *x* and *y*.

Here we find the derivative on both sides of the equation while treating *y* as a function if *x*.

$3x^2 + 3y^2y' = 6xy' + 6y$

Note the use of the chain rule on the left side when differentiating $3y^3$ and the use of the product rule on the right side

At this point we try to solve for *y*'.

$3y^2y' - 6xy' = 6y - 3x^2$

$y'(3y^2 - 6x) = 6y - 3x^2$

$$y' = \frac{6y - 3x^2}{3y^2 - 6x} = \frac{2y - x^2}{y^2 - 2x}$$

So now we have an expression for the derivative of the curve at each point in terms of *x* and *y* as we desired.

If we wanted to know the equation of a line tangent at some point on this curve, say at (3,3)

Note that $3^3 + 3^3 = 54 = 6(3)(3)$

We have
$$y' = \frac{2y - x^2}{y^2 - 2x} = \frac{6-9}{9-6} = -1$$

Using $y = -x + b$ we find that b=6 so the equation of the tangent line is $y = -x + 6$.

Example:

Find y' if $sin(x+y) = y^2 cos(x)$

On the left we have $sin(x+y)' = cos(x+y)(1+y')$

On the right $(y^2 cos(x))' = y^2(-sin(x)) + 2yy'cos(x)$

On the left we have $cos(x+y)(1+y') = y^2(-sin(x)) + 2yy'\,cos(x)$

Solving for y' we get

$$cos(x+y) + cos(x+y)y' = y^2(-sin(x)) + 2yy'cos(x)$$

$$cos(x+y)y' - 2yy'cos(x) = y^2(-sin(x)) - cos(x+y)$$

$$y'\,(cos(x+y) - 2y\,cos(x)) = y^2(-sin(x)) - cos(x+y)$$

$$y' = \frac{y^2(-sin(x)) - cos(x+y)}{cos(x+y) - 2y\,cos(x)} = \frac{y^2(sin(x)) + cos(x+y)}{2y\,cos(x) - cos(x+y)}$$

Some in Class Examples:

Exercises

Find dy/dx by implicit differentiation.

1.* $x^2 + y^2 = r^2$.

2. $x^3 + y^3 - 3axy = 0$.

3.* $b^2x^2 + a^2y^2 = a^2b^2$.

4. $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

5.* $x^{2/3} + y^{2/3} = a^{2/3}$.

6. $y^2 = 4cx$.

7.* $x^4 + 4x^3y + y^4 = 1$.

8. $(2y)^{1/2} + (3y)^{1/3} = x$.

9.* $x + 2xy + y = 1$.

10. $x^2 + axy + y^2 = b^2$.

Find the slope at the indicated point.

11.* $2x + 3y = 5$; $(-2, 3)$.

12. $9x^2 + 4y^2 = 72$; $(2, 3)$.

13.* $x^2 + xy + 2y^2 = 28$; $(-2, -3)$.

14. $x^3 - axy + 3ay^2 = 3a^3$; (a, a).