Math 109 Calc 1 Lecture 6

Derivative Formulas

Section 3.1
Simplifying how we find Derivatives

In our last class we found the derivatives of a few basic functions

For $f(x) = c$ we have $f'(x) = 0$

For $f(x) = mx + b$ we have $f'(x) = m$

For $f(x) = x^2$ we have $f'(x) = 2x$

For $f(x) = \sqrt{x}$ we have $f'(x) = \dfrac{1}{2\sqrt{x}}$

And For $f(x) = \sqrt{1 - x^2}$ we have $f'(x) = \frac{-x}{\sqrt{1-x^2}}$

It would be very tedious to have to use the derivative formula

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

every time we need to find a derivative.
So here we drive some general formulas or rules which make such calculations quite simple.

The derivative of a sum is the sum of a derivative

1. $(\boldsymbol{f} + \boldsymbol{g})'(\boldsymbol{x}) = \boldsymbol{f}'(\boldsymbol{x}) + \boldsymbol{g}'(\boldsymbol{x})$

Proof

$$(f + g)'(x) = \lim_{h \to 0} \frac{(f + g)(x + h) - (f + g)(x)}{h} =$$

$$\lim_{h \to 0} \frac{f(x + h) + g(x + h) - [f(x) + g(x)]}{h} =$$

$$\lim_{h \to 0} \frac{f(x + h) - f(x) + g(x + h) - g(x)}{h} =$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}=$$

$f'(x)+g'(x)$
The derivative of a constant times a function is the constant time the derivative
2. $(\boldsymbol{Kf})'(\boldsymbol{x}) = \boldsymbol{Kf}'(\boldsymbol{x})$

Proof

$$(Kf)'(x)=\lim_{h\to 0}\frac{Kf(x+h)-Kf(x)}{h}= K\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=Kf'(x)$$

The Product Rule
3. $(\boldsymbol{f}\cdot\boldsymbol{g})'(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x})\cdot\boldsymbol{g}'(\boldsymbol{x})+\boldsymbol{g}(\boldsymbol{x})\cdot\boldsymbol{f}'(\boldsymbol{x})$

Proof

$$(f\cdot g)'(x)=\lim_{h\to 0}\frac{(f\cdot g)(x+h)-(f\cdot g)(x)}{h}=$$

$$\lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h}=$$

Now we pull a rabbit out of our hat.

We add and subtract $f(x+h)g(x)$from the numerator.

$$\lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)+f(x+h)g(x)-f(x+h)g(x)}{h}=$$

We rearrange and take out $f(x+h)$ as a factor and also take out $g(x)$ as a factor

$$\lim_{h\to 0}f(x+h)\frac{g(x+h)-g(x)}{h}+\lim_{h\to 0}g(x)\frac{f(x+h)-f(x)}{h}=$$

Since $f(x)$ is continuous at x, this becomes

$$f(x)\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}+g(x)\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=$$

$$f(x)\cdot g'(x)+g(x)\cdot f'(x)$$

Using this theorem, it is easy to find the **Power Rule** that if *n* is an integer > 0

4. $\boldsymbol{(x^n)' = n(x^{n-1})}$

We do this with a proof by induction.

For such a proof we have a series of statements *P*(*n*) where *n*=1,2,3…
We need to show two things for a proof by induction

1. The anchor is true *P*(1)
2. if we assume *P*(*n*) then *P*(*n*+1) is true.

Proof

P(1) is the statement $\boldsymbol{(x^1)' = (x^0) = 1}$

This is a simplification of
For $f(x) = mx + b$ we have $f'(x) = m$

Assuming that *P*(*n*) or $(x^n)' = n(x^{n-1})$ is true

we want to show that *P*(*n*+1) or $(x^{n+1})' = (n+1)x^n$ is true.

We do this using the product rule

$$(x^{n+1})' = (x \cdot x^n)' = x(x^n)' + (x)'x^n = x \cdot nx^{n-1} + x^n = nx^n + x^n = (n+1)x^n$$

Example

If $P(x) = 12x^3 - 6x - 2\ what\ is\ P(x)'$?

$$P(x)' = 12 \cdot 3x^2 - 6 = 36x^2 - 6$$

If $P(x) = \frac{1}{4}x^4 - 2x^2 + x + 5\ what\ is\ P(x)'$?

$$P(x)' = \frac{1}{4} 4 \cdot x^3 - 2 \cdot 2x + 1 = x^3 - 4x + 1$$